Chapter 3
The Mathematical Capital and Its Economic Value

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3.1 Introduction

At all the times, knowledge, research, technology, innovations and entrepreneurship have been the "engines" driving the whole process of economic and social evolution.

The economic growth on a global scale mainly depends upon the diffusion of knowledge concerning new techniques of production and management. Recognizing the growing significance of the knowledge-based economy, the OECD (1996) defines the latter as one "directly based on the production, distribution and use of knowledge and information," concluding that "knowledge and information tend to be abundant; what is scarce is the capacity to use them in meaningful ways." While all these are correct, it is also true that knowledge only reaches its full potential in creating economic value when it becomes embedded in organizational processes and routines.

Unlike physical goods that are consumed as they are used providing decreasing returns over time, knowledge, by contrast, provides increasing returns as it is used (Romer 1992). The more knowledge is used, the more valuable it becomes, creating thereby a self-reinforcing cycle. Stiglitz (1999) characteristically emphasizes that "it is the process of embodying knowledge in people (learning) and things (applications) that is costly in time and resources."

The "new economy of knowledge" is defined as possessing the following characteristics: highly intensive knowledge; rapid diffusion of information; extensive innovation networks; high levels of education, skill and training; as well as firms' linkages across complementary assets and competencies (Marceau et al. 1997). In this context, it is the efficient utilization of knowledge resources that offers a competitive advantage, and, in addition, there is need for a continuously increasing share of "intellectual workers" in labour force.

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During the last six decades, the contribution of technology, human capital, research, innovation and entrepreneurship on economic growth has been theoretically defended and empirically verified (Solow 1957; Abramovitz 1962; Schultz 1961, 1963; Becker 1964; Lucas 1988; Romer 1992). In addition, the impact of education on production, accumulation and diffusion of human capital, as well as on productivity, earnings, income distribution and social cohesion has also been theoretically and empirically validated and estimated (Schultz 1961, 1963; Becker 1964; Mincer 1974; Psacharopoulos 1994), but also critically debated (Bowen 1964; Arrow 1973; Spence 1973).

Throughout centuries, the role of mathematics in education, training and scientific research has been central. Mathematics operated as a catalyst for a very large number of communication processes among natural and/or social disciplines, engineering sciences and arts. After the Second World War, in a world driven by numbers, the scientific research has been increasingly using mathematical knowledge, while the new high technology has also embodied it. In fact, mathematics appears to pervade every study and technique in our modern world.

In turn, other sciences, having borrowed and employed the methods and "tools" of mathematics, pose new problems and generate new sources of inspiration for mathematics. Furthermore, technology has had a profound effect on mathematics. More specifically, the mathematical knowledge/technology constitute the core of the new knowledge economy—the so-called digital economy. Thus, it may be argued that the mathematical research, knowledge, education and technology are determining factors for the economic, social, cultural and political evolution of individuals, social groups and countries.

Yet, the development of mathematical research (in pure and applied mathematics), mathematical education and mathematical applications requires a share of the already insufficient private and social resources, which in turn raises direct questions with clear economic implications: How much mathematics does the economy and society need as consumption and investment[1]? How much mathematical research (in pure and applied mathematics) and mathematical technology is needed? How much and which mathematics is needed at the different levels and directions of education and training?

The purpose of this chapter is to introduce the concept of the individual and social mathematical capital and delineate its economic value, as well as detect a string of related issues open to discussion. Addressing these issues is a prerequisite for the optimization of planning concerning the procedures of production, accumulation and diffusion of mathematical capital and, by implication, for the more efficient contribution of mathematical capital to economic, social and cultural evolution.

The structure of this chapter is the following. Section 2 provides a brief answer to the question of "what is mathematics?" Section 3 introduces the concept of mathematical capital and its economic value. Section 4 presents a series of problems open to discussion. The last section offers some concluding comments.

[1] Consumption and investment are important concepts in economics. Whereas consumption refers to the purchase or use of goods and services which bring immediate but short-lived benefits, investment refers to the acquisition of assets which yield benefits over a long period of time.

3.2 What Is Mathematics ?

What is mathematics? How was it created? By whom has it been developed and practised? What is its role in the history of scientific thinking? What is its relationship with other disciplines? These are some of the old-standing questions that have been subject to intense discussion and to which, in the course of the evolving debate, new answers have been offered.

The word "mathematics" comes from the Greek "mathema" (sixth to third century BC), which means learning, study and science. Since the classical times, however, it came to have a narrower and more technical meaning as "mathematical study." Considering the richness of the definition of "mathematics" from the ancient to modern times, it is not prudent, for the purposes of this chapter, to attempt to give a single, precise and comprehensive definition. Nonetheless, it is sufficient to note that, contrary to a widespread opinion among non-scientists, mathematics is not a closed and perfect edifice. While it constitutes an autonomous and independent scientific field, mathematics also has the potentiality of mirroring and modelling all processes of thought and, perhaps, of sciences. In other words, mathematics represents an "extrovert" so to speak body of knowledge. In this context, it continuously collaborates with observational and laboratory sciences, yet, through its power to quantify and organize knowledge, it makes possible the application of such knowledge to problems extending over a vastly larger scope than that of the aforementioned sciences. One could even go so far as to say that mathematics was necessary for man's conquest of nature and for the development of the human race through the shaping of the modes of thinking.

Like other sciences, mathematics has been subject to great changes during the past 60 years. Not only has its scope vastly increased and not only has the emphasis on what were considered the central problems changed but also the tone and the aims of mathematics have to some extent been transmuted. There is no doubt that many great triumphs of physics, astronomy, biology, economics and other sciences arose, to a significant extent, from mathematics. Since the sixth century BC, mathematics has expanded and there has been a fruitful interaction with other sciences, to the benefit of both.

Broadly speaking, mathematics can be subdivided into the study of quantity, structure, space and change. It is the natural language of sciences and engineering, for example, and at the same time forms an essential tool for business, industry and generally for the "information revolution." In addition to these main classifications, there are also subgroupings focusing on the exploration of links between the core of mathematics and other fields, including logic, set theory, empirical mathematics of various sciences and, more recently, rigorous study of uncertainty.

In conclusion, mathematics is a field under continuous development and in constant search of new applications. Indeed, mathematics is deeply interconnected with modern life.

3.3 Mathematical Capital and Its Economic Value

In economic theory, "physical capital" or just "capital" refers to any manufactured asset that is applied to production, such as machinery and buildings. The "capital" is one of the three primary factors of production, with the other two being "natural resources" and "labour." All these three constitute the initial inputs in the production function.

Smith (1776) first pointed out that education helped to increase the productive capacity of workers, in the same way as the purchase of new machinery or other forms of physical capital. At the beginning of 1960s, the concept of human capital was introduced for the first time, and the human capital theory was shaped as an independent trend in the neoclassical economic theory (Schultz 1961, 1963; Becker 1964, etc.). Since then it constitutes new input in the production function.

Human capital refers to the stock of competences, knowledge, skills and personality attributes that are embodied in the ability to perform labour in order to produce economic value, and can be infinitely elastic, encompassing non-measurable variables, such as the personal character or relations with insiders. The concept of human capital was initially approached by Schultz (1961) and Denison (1962) and later on as "stock" by Lin (2003) or "flow" by Mankiw et al. (1992). The human capital theory suggests that education and training are the most important investments in human capital. All in all, mathematical capital appears to constitute the basic component of human capital.

Consequently, by analogy, "mathematical capital of an individual" may be construed as encompassing all inherent and acquired mathematical abilities, all acquired mathematical knowledge (logic, foundations and structure, critical thought, methodologies, techniques), skills, experiences and effectiveness in mathematical applications. By extension, "mathematical capital of a social group" may be interpreted as covering the sum of the overall mathematical capital of the social group's members and the mathematical tradition and culture of the group. The inherent mathematical ability/skill/talent is one of the seven attributes composing the "spectrum of human's intelligence" (Gardner 1993). The so-called "acquired abilities" are derived through the processes of education, research and experience.

The framework of hypotheses concerning the economic characteristics of mathematical capital and its function within the economy can be summarized as follows:

1. The research in pure and applied mathematics is the main mechanism of production accumulation and diffusion of the individual/social mathematical capital.
2. The education in mathematics is the main, institutional mechanism of diffusion, accumulation and production of the individual/social mathematical capital.
3. The rate of production and accumulation of the acquired part of the mathematical capital for each individual and social group depends, mainly, on the mathematical tradition; on the social, economic and cultural environment; and on the quality of research, education and training systems. The exploitation of mathematical capital occurs at different levels of effectiveness.
4. The research and education in mathematics can be considered as consumption but mainly as investment for both the individual and the society.[1] The consump-

tion and the investment element provide utility (now or later) and contribute to the discounted stream of utility enjoyed by the economic agent. The production and accumulation of the mathematical capital from every individual and/or society require investment in both time and economic resources. An individual or family decision unit selects an amount of investment in mathematical capital (or other self-investment) in order to maximize an objective function subject to some constraints. The objective function may be the lifetime income, appropriately discounted, or it may be a utility (i.e. a measure of well-being). The constraints include the limits imposed by a family's own financial resources, its capacity to borrow outside funds and the limits upon the time the individual (and, in earlier years, the parents as well) can devote to mathematical education or research. Costs include out-of-pocket payments plus earnings forgone. Benefits include the increase in expected lifetime earnings, as well as non-pecuniary returns, such as improved working conditions and job security. Optimal investment in mathematical capital occurs when the discounted value of the costs incurred equals the discounted value of the benefits expected. A society selects an amount of investment in mathematical capital in order to maximize an objective social welfare function subject to some constraints. Optimal social investment to mathematical capital occurs when the discounted social cost incurred equals the discounted value of the benefits expected.

5. Despite its long-term character, the investment in mathematical capital is accompanied by limited uncertainty (estimation methods: sensitivity analysis, risk analysis or new method) due to its continuing usefulness measured on both individual and social basis.
6. Expenditure on mathematical capital can be classified as either consumption or investment, although the borderline is not always precise.
7. The individual and social investment in mathematical capital is progressively reduced during an individual's life, tending to zero at the stage of retirement from work. Other things being equal, the earlier the investment, the longer will be the expected stream of benefits. Hence, maximization of lifetime utility implies that most formal mathematical education and training will occur during one's youth. The return on mathematical capital has a long-term character.
8. The mathematical capital reserves/stock borne by each individual cannot be transferred, cannot be bought and cannot be inherited in its entirety.
9. The quantity and quality of mathematical capital contribute to the improvement of individual productivity and to the total productivity of society. The mathematical capital increases productivity through the following basic channels: (a) it enhances the ability of an individual to perform standard tasks (factor efficiency) and learn to perform new tasks; (b) it improves the ability to receive and process new information; (c) it develops the ability to evaluate and adjust to changing circumstances (ability to deal with disequilibria); (d) it increases the ability to communicate and coordinate activities with one another; (e) it reduces the subjective uncertainty and unnecessary anxiety, on the one hand, whereas, on the other hand, it boosts the establishment of a more critical approach towards the status quo, fostering, in parallel, the probability of accepting new technologies and practises; and (f) it helps to

bring about new innovations in production processes and to develop new products.
10. The mathematical capital is a resource which increases rather than diminishes with use. It has little cost to generate and diffuse. Once mathematical knowledge is discovered and made public, there is essentially little marginal cost to adding more users.
11. The share of mathematical capital which is not used diminishes with the passage of time, or as the individual grows older, or with evolution and changes in sciences, technology and production. The mathematical capital which is not continuously replenished is to be quickly diminished.
12. The differences in quantity and quality of individual and social mathematical capital affect the inequities between individuals and societies and determine, to a significant extent, the hierarchies.
13. The benefits from mathematical research and education can be classified into private (accruing to the individual and the family) and social in both a direct-narrow (accruing to society at large) and indirect-wide (including externalities[2] and spillovers[3]) sense. The benefits could also be categorized into micro-level (monetary/private returns on investment in mathematical research and education and social returns in narrow sense) and macro-level (market and non-market, social benefits in wide sense, positive externalities and spillovers).
14. Mathematical capital embodied in new products and services has become a source of wealth creation.
15. The mathematical capital contributes to economic growth and development, and vice versa, that is, economic development contributes to mathematical capital development.

3.4 Open Problems for Evaluation and Planning

In order to achieve the optimal planning of policies and investments regarding the processes of production, accumulation and diffusion of mathematical capital, it is necessary to address the following problems:

1. The construction of methods and models to proxy for the quantity and quality of mathematical capital
2. The determination of private and social benefits (markets, non-markets, externalities, spillovers) derived from mathematical capital
3. The measurement of private and social cost of production, accumulation and diffusion of mathematical capital

[2] Externalities (positive) are those benefits to society that are above and beyond the private benefits realized by the individual "decision-maker," that is, the student or the family. These benefits are above and beyond private monetary and non-monetary consumption benefits, both of which are captured by the "decision-maker" and taken into account whenever the decision is made.

[3] Spillovers are a type of externality, since they are a benefit that is not captured by the decision-making unit within which occurs the mathematical educational process.

4. The estimation of the impact of mathematical capital on earnings of workers
5. The estimation of the return rates of private and social investments in mathematical capital (methods: cost–benefit analysis or new methods)
6. The economic and social evaluation of investment and policies in both mathematical research (pure and applied mathematics) and education (methods: cost-benefit analysis, cost-effectiveness analysis or new methods)
7. The assessment of public expenditures on mathematical capital (methods: cost-effectiveness analysis or new methods)
8. The estimation of mathematical capital contribution to economic growth/development/welfare, income distribution and social cohesion (methods: with models of new growth theory or new models)
9. Determining whether, in the interaction process between economic development and mathematical capital development, the latter comes first

3.5 Concluding Comments

This chapter highlighted the pivotal role of mathematical capital in the scientific research, education, applications and, more generally, new knowledge-based economy. The concept of individual and social mathematical capital was defined. The skeleton of the mathematical capital's economic characteristics was delineated, and its importance as an economic resource was stressed. The process of production, accumulation, transmission and diffusion of mathematical capital in economy and society was highlighted. Finally, a series of open problems was raised which deserve the attention of economists and mathematicians.

The economic and social evaluation of policies and investments concerning the production, accumulation and diffusion of mathematical capital leads to the efficient planning for the optimal use of scarce resources. It follows, therefore, from the discussion of this chapter that the mathematical capital has to be recognized and utilized accordingly by the scientific, economic, social and political elite as a strategically important resource.

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